Bat Algorithm based Optimization of a Multiuser Cognitive Radio System over Nakagami- m Fading Channels

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Abstract: Optimization of an ideal cognitive radio system(without channel impairments) with single secondary user has been successfully carried in past by other and current authors. This communication is a detailed extension to it, in which optimization of a multiuser cognitive radio system in a dynamic fading environment, specifically Nakagami-m fading has been introduced. The fitness functions have been modified to include the fading effects. Bat algorithm is then applied to optimize the transmission parameters of multiple secondary users. IEEE 802.22 WRAN standard has been followed to decide the range of all these parameters. Results obtained by Bat algorithm are compared with the onesprocured by GA, SA and BBO to show Bat algorithm's upper hand in cognitive radio optimization.

Keywords: Cognitive Radio, Nakagami-m fading, Optimization, Bat algorithm, Fitness Function

Introduction

Cognitive radio (CR) has made a mark in spectrum world by acting as a solution to the problem of spectrum scarcity. It has allowed the efficient utilization of the spectrum by wise allocation of available licensed frequency slots to various unlicensed wireless applications and services [1-3]. The ability of CR to perceive, grasp and modify smartly in the environment has offered such an optimization of services, according to the needs of user. Up to date many search heuristic techniques like genetic algorithm (GA) [4] and ant colony optimization (ACO) [5] have been used for optimization of CR system [6].
Many efficient search algorithms have been evolved in the later years to perform optimization in every possible field from engineering to business planning, including the traditional simulated annealing (SA) [7], efficient biogeography-based optimization (BBO) [8] and recently developed Bat algorithm (BA) [9].
Present authors have applied SA and BBO for design optimization of a single user CR system without fading and considered them to be better than other explored evolutionary algorithms [10], [11].
Bat algorithm is a relatively more recently developed global optimization technique by Xin-She Yang [12]. It is a swarm based metaheuristic algorithm whose formation is encouraged by echolocation conduct of bats. Even though having an intricate implementation procedure, it has outshined other commonly used techniques such as harmony search, particle swarm optimization and genetic algorithm. Echolocation based ingenious amalgamation of majority of good attributes of these techniques has led to development of a more balanced and successful optimization algorithm, namely bat algorithm.
The bat algorithm has been applied to a new CR system, where a licensed band channel (TV channel) allocated to primary user is affected by Nakagami-m fading [13]. This fading affected channel is being shared by a number of secondary users (or cognitive users), while trying to avoid interference in between them. The optimal transmission parameters of cognitive users have been achieved by forming multi-objective fitness functions to achieve desired quality of service (QoS) in terms of minimum transmit power, minimum BER, minimum interference, maximum spectral efficiency and minimum entropy with the outage probability constraint [14], under the effect of fading [15].
Section 2 of paper gives a brief description about used optimization technique, bat algorithm. Section 3 discusses new CR system and provides mathematical formulations and analytical expressions of probability density functions (pdf) required for signal to interference plus noise ratio (SINR) of primary and secondary users. Formulation of fitness functions along with a look on objectives to be full filled is also described in this section. Followed by this in section 4 is the simulation and results for optimization of CR system using bat algorithm. The work has been concluded in the last.

Bat Algorithm

Bat algorithm refers to an optimization technique which provides optimum solution to any problem by capitalizing echolocation characteristic of bats [12]. Echolocation points towards the biological sound propagation technique used for navigation, communication and detection. Many animals like bats, whales and dolphins use echolocating, i.e. they send calls to the environment and uses the echoes of their calls to discover and recognize the objects. The three rules that govern the bat algorithm by using echolocation as the basic principle are:

- Echolocation is used as the basic block by all the bats to feel and notice distance. They are also capable of distinguishing between their food (prey) and environmental obstacles in a strange secret way.
- In the lurch of their hunt, bats move randomly with certain velocityv_p at a position x_p with constant frequency f_{min}, changing wavelength λ and loudness A_0. According to the range (closeness) of their aim, bats automatically change the wavelength and rate of pulses emitted.
- Loudness is presumed to be varying from a large (positive) $A_o$to a minimum fixed value A_{min}.

Step by step process of bat algorithm is given as:

1. This algorithm begins with arbitrary initialization of the bat population of (say) *NP* vectors, each of *n* dimensions.

$$X_{ij} = X_{\min j} + rand(X_{\max j} - X_{\min j});\ \text{for } i = 1,2,3\ldots NP;\ j = 1,2,3\ldots D \qquad (1)$$

Where X_{min} and X_{max} are lower and upper bounds for j^{th}component or decision variable.

2. Calculate the potential (fitness) for each possible solution in population *NP*.
3. Initialize the echolocation parameters, i.e. pulse frequency f_p for a location x_p, loudness A_p and pulse rate r_p.
4. Until a termination criteria is reached (maximum iterations), repeat the steps 4 to 9.
5. Evaluate and update the velocity and position of each bat using the equations:

$$v_p^m = v_p^{m-1} + f_p\left(x_p^m - \bar{x}\right) \qquad (2)$$

$$x_p^m = x_p^{m-1} + v_p^m \qquad (3)$$

$$f_p = f_{min} + \beta^0(f_{max} - f_{min}) \qquad (4)$$

Where v_p^m and x_p^m denotes new velocity and position of p^{th} vector at time step m respectively. $\bar{x}$is the present global solution (best location). $f_p$is the frequency of selected p^{th}vector, which is calculated from minimum ($f_{min} = 0$) and maximum ($f_{max} = 2$) values of frequency. $\beta^0$is an arbitrary vector found from uniform distribution, lying in the range of 0 to 1.

6. The fitness for each bat is again determined after position and velocity updating. The best solution among current population is selected.
7. Near this selected best solution, a local solution is generated by using the concept of random walk.

$$x_{new} = x_{previous} + \epsilon A^m \qquad (5)$$

Where$A^m$is the average loudness value of all bats and ϵ is is random number (0 to 1), which indicates theintensity and direction of random-walk.

8. Accept locally created solutions. Increase pulse rate and decrease loudness.
9. The local best solution after random walk is ranked along with other selected best solutions on the basis of their fitness value. Best among them is finally selected as global best, referred to as optimum solution.
10. The process continues until a termination criterion is met.

Cognitive Radio System

1. **Cognitive Radio Model in Nakagami-m Fading:** A primary user (PU) is presumed to be using the allocated licensed channel, which in this case is a TV channel in VHF band for transmission of data in rural areas (sparsely populated areas) [16]. This available TV white space allows multiple (*k*) secondary users (SUs) to opportunistically access the channel for various broadband services on non-interfering basis [17]. The system set up for the above model is illustrated in Fig. 1.

The transmitted power of the PU is constant and is denoted by P_P. The variable transmitted power of n^{th} SU, which is one of the parameters to be optimized, is denoted by P_{sn}. The respective SINR of PU and SU are given by γ_p and γ_s. PU and SU use the channels experiencing Nakagami-m fading, hence the power gains of both the channels follow Gamma-m distributions. The pdf of primary user's channel gain is expressed as $f(x)$ and that of secondary user's is$f(y)$ and both are given as:-

$$f(x) = \frac{1}{\Gamma(m_p)}\left(\frac{m_p}{P_p}\right)^{m_p} x^{m_p-1}\, exp\left(\frac{-m_p}{P_p}x\right) \qquad (6)$$

$$f(y_n) = \frac{1}{\Gamma(m_{sn})}\left(\frac{m_{sn}}{P_{sn}}\right)^{m_{sn}} y_n{}^{m_{sn}-1}\, exp\left(\frac{-m_{sn}}{P_{sn}}y_n\right) \qquad (7)$$

Where n is number of secondary users and m_p and m_s are called the shape parameters of the primary and secondary users.Now for calculating SINR mathematically; the expected values of pdf of primary and secondary channels are found as [18]:

$$\mathrm{E(x)} = \mathrm{E}\big(\mathrm{f(x)}\big) = \int_0^\infty x.f(x)dx = \int_0^\infty x.\frac{1}{\Gamma(m_p)}\left(\frac{m_p}{P_p}\right)^{m_p} x^{m_p-1}\ exp\left(\frac{-m_p}{P_p}x\right)dx \qquad (8)$$

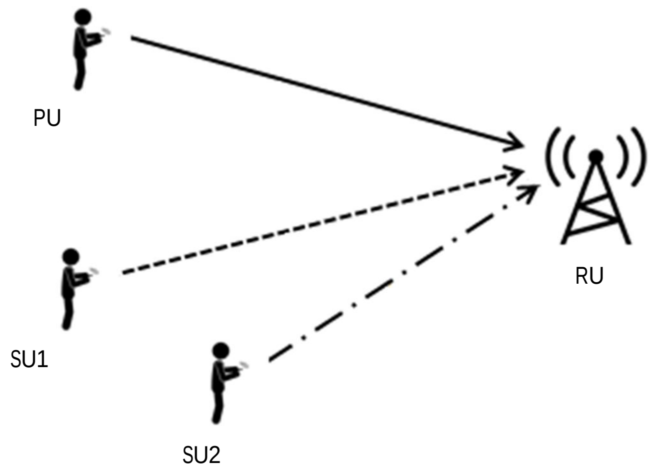

Figure 1. CR system model

Where $\Gamma(\,)$ is standard gamma function

$$E(x) = \frac{1}{\Gamma(m_p)}\left(\frac{m_p}{P_p}\right)^{m_p}\int_0^\infty x^{m_p}\, exp\left(\frac{-m_p}{P_p}x\right) = \frac{1}{\Gamma(m_p)}\left(\frac{m_p}{P_p}\right)^{m_p}\left(\frac{m_p}{P_p}\right)^{-m_p-1}\Gamma(m_p+1) = \left(\frac{P_p}{m_p}\right)\frac{\Gamma(m_p+1)}{\Gamma(m_p)} \quad (9)$$

Similarly for n^{th} secondary user, pdf can be expressed as:

$$\mathrm{E}(y_n) = \mathrm{E}(\mathrm{f}(y_n)) = \int_0^\infty y_n . f(y_n) dy_n = \int_0^\infty y_n . \frac{1}{\Gamma(m_{sn})}\left(\frac{m_{sn}}{P_{sn}}\right)^{m_{sk}} {y_k}^{m_{sn}-1} exp\left(\frac{-m_{sn}}{P_{sn}}y_n\right) dy_n = \left(\frac{P_{sn}}{m_{sn}}\right)\frac{\Gamma(m_{sn}+1)}{\Gamma(m_{sn})} \quad (10)$$

Now the SINR of the PU is given as:

$$SINR_p = \gamma_p = \frac{P_P C_P}{\sum_{i=1}^{k} P_{si}C_{si}+\sigma^2} = \frac{P_P E(x)}{\sum_{i=1}^{k} P_{si}E(y_i)+\sigma^2} \quad (11)$$

Where C_P and C_S are channel gains of PU and SU respectively and σ^2represent additive white Gaussian noise with a variance (σ^2) of 0.1 at receiver unit.Using the values of $E(x)$and $\mathrm{E}(y_n)$ found in equation (9) and equation (10), $\mathrm{SINR_p}$is expressed as:

$$SINR_p(\gamma_p) = \frac{P_P\left(\frac{P_p}{m_p}\right)\frac{\Gamma(m_p+1)}{\Gamma(m_p)}}{\sum P_{sn}\left(\frac{P_{sn}}{m_{sn}}\right)\frac{\Gamma(m_{sn}+1)}{\Gamma(m_{sn})}+\sigma^2} \quad (12)$$

Accordingly, SINR of k^{th} the secondary user, denoted byγ_{sK}, is given as:

$$SINR_{sk}(\gamma_{sk}) = \frac{P_{sK}E(y_k)}{P_P E(x)+\left[\sum_{i=1}^{n} P_{si}E(y_i)-P_{sk}E(y_k)\right]+\sigma^2};\ k < n \quad (13)$$

2. **Cognitive Radio Parameters and Objectives:** The CR system, owing to its real time interaction with the environment is capable of determining the optimized combination of transmission parameters to meet the desired QoS. The adjustable transmission parameters used in bat algorithm are Transmitted Power (P), Modulation Type, Modulation Index (M), Bandwidth (B), Time Division Duplexing (TDD) and Symbol Rate (Rs). Bit-error-rate (BER), signal-to-noise ratio (SNR) and the noise power (N) are the considered environmental parameters in this work. They provide information about the surrounding environment to the CR system.

Five different objectives are laid down to assist CR system in satisfying its intended goals. Their fitness function have been defined in the below section, in order to move the system towards an optimal region.

3. **Formulation of Fitness Functions for Cognitive Radio System:** Optimization the above enlisted transmission parameters of SUs is done in order to achieve the desired QoS, which is defined by the five objectives.

1. Minimize power utilization: Reduce the quantity of power to be utilized by the system.

$$f_{\min power} = \frac{1}{N_c P_{smax}}\sum_{i=1}^{N_c} P_{si} \quad (14)$$

Where N_c represents total number of channels that SU operates on, P_{si} is power transmitted by each secondary user and P_{smax} denotes the maximum power transmitted by the user.

2. Minimum BER: Enhance the overall BER of communication system.

$$f_{\min BER} = \frac{\log_{10}(0.5)}{\log_{10}(\overline{P_{be}})} \quad (15)$$

Where $\overline{P_{be}}$ is the average BER over N_c channels by n SUs and $P_{be}$for a single M-ary QAM channel is given as:

$$P_{be} = \frac{4}{\log_2(M)}\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3\log_2(M)}{M-1}\gamma_s}\right) \tag{16}$$

Where γ_s is SINR of each SU affected by Nakagami-m fading, which is given in equation (13). Fulfillment of this objective is done subjective to the primary outage probability constraint. As while trying to minimize BER of SU, transmitted power of PU will decrease, causing a risk of outage to PU. Hence $\gamma_P > \gamma_0$; where $\gamma_0$is the minimum SINR required by PU and γ_P is given by equation (12).

3. Maximum Throughput: Increase the overall data throughput transmitted by the radio.

$$f_{\max throughput} = 1 - \frac{\sum_{i=1}^{n}\frac{log2(M_i)}{log2(M_{max})}}{N_c} \tag{17}$$

4. Minimum Interference: Reduce the radio's interference contributions.

$$f_{\min interference} = \frac{\sum_{i=1}^{n}((P_{si}+B_i+TDD_i)-(P_{smin}+B_{min}+1))}{N_c(P_{smax}+B_{max}+100)} \tag{18}$$

Where TDD_i refers to time division duplexing, which defines the time consumed for transmission by each SU.B_i is the bandwidth required for a single user, B_{min} and B_{max} is the minimum and maximum bandwidth available.

5. Maximum Spectral Efficiency: Increase the optimal use of the frequency spectrum.

$$f_{\max speceff} = 1 - \frac{\sum_{i=1}^{n}\frac{M_i \times R_i \times B_{min}}{B_i \times M_{max} \times R_{max}}}{N_c} \tag{19}$$

Where M_i is is the modulation index of a single secondary user, and M_{max} is the maximum modulation index. R_i is the symbol rate and R_{max} is the maximum symbol rate.

To fulfill these five multiple objectives, the normalized weighted sum approach is used, where the sum of weights is limited to 1. It simplifies the problem by combining multi objectives into oneobjective function. This transformation is shown in equation (20), where a single value is obtained by adding the multiples of each objective and a weight vector *wn*.

Then the final combined objective is given as:

$$f_{five\text{-}objective=}\ w1*(f_{min\text{-}power})+w2*(f_{min\text{-}ber})+w3*(f_{max\text{-}tp})+w4*(f_{min\text{-}intr})+w5*(f_{max\text{-}spec\ eff}) \tag{20}$$

The values of the weights, w1, w2, w3, w4, and w5 guide the search in a particular direction for the optimizing bat algorithm. Five weight vectors generate five different scenarios for a CR system. Table 1 depicts the required values of these weights to emphasize the search in the desired direction for the specific scenario.

Simulations and Results

Optimization of basic CR system without fading has been carried on earlier by Newman using Genetic Algorithm (GA) [4] and by present authors using SA [10] and BBO [11]. The same setup of CR system including same transmission parameters, objectives and related weights has been used by bat algorithm in section 4.1, to ensure the finer behavior of bat algorithm over the earlier used optimization techniques. Then section 4.2 discusses the optimization of multiuser fading affected CR system using bat algorithm.

1. **Simulation for Single carrier Basic CR System:** In this section bat algorithm has been applied to a single user ideal CR system (without fading) to fulfill the five objectives that have been discussed in [10] and [11]. Values of transmission parameters and weight vectors required for multiobjective optimization has also been kept same to ensure the fair comparison among bat algorithm, BBO, SA and GA. The parameters of these three optimization techniques have also been kept intact as given in [10] and [11].

The important parameters used in bat algorithm includes Population size (*NP*) = 10, loudness (A_p) = 0.5 and pulse rate (r_p) = 0.5. The number of iterations of bat algorithm has been restricted to 200, same as used by BBO in previous work. Table 2 shows the results obtained on basic CR system using bat algorithm. The bold values of the objectives fulfilled represents that the particular objective has been optimized successfully in its corresponding mode.

Table 3 shows the comparison between bat algorithm, BBO, SA and GA which have been applied on the same problem in the literature. As optimization of CR system is considered to be a minimizing problem, therefore decreasing fitness score for bat algorithm is itself a indicator of its better performance. Even the values of the objectives obtained by using the hybrid are also more optimized compared to those obtained by other three techniques.

2. **Simulation for Multiuser Fading Affected CR system:** In this section bat algorithm has been applied to a multiuser CR system to fulfill the five objectives discussed above using MATLAB 7.10. The CR system considered is operating in TV white space and using IEEE 802.22 standard. It is the first such wireless regional area network standard which allows cognitive radio to share and efficiently utilize the vacant frequency slots of television broadcast service, avoiding interference. The ranges of the parameters used in CR system are taken according to this standard. The transmitted power (P) varies between 0.1 and 2.5 mW. QAM modulation scheme is used with number of symbols (M) ranging from 16 to 64.

Bandwidth (B) available is from 6 to 8 MHz. TDD percentage is taken between 25 and 100% and Symbol rate (Rs) between 2.25 and 5.16 Msps.

The important parameters used in bat algorithm are kept to be same as discussed in section 4.1 for single carrier basic CR system. Only two secondary users (n=2) along with one primary user are considered for the convenience in simulation. The Table 4 shows the optimized values of the five parameters for both the SUs, along with their respective fitness value obtained by using bat algorithm. Five different scenarios are explored, each giving maximum weightage to one of the objectives. Table 5 shows the values obtained for each objective and the values of the desired objectives that we want to achieve in that particular scenario are bold.

Table 1.Weightingscenarios for five objectives

Scenario	Weight Vector [$w1$, $w2$, $w3$, $w4$, $w5$]
Minimum power mode	[0.45 0.10 0.20 0.15 0.10]
Minimum BER mode	[0.10 0.50 0.10 0.10 0.20]
Maximum throughput mode	[0.10 0.15 0.50 0.15 0.10
Minimum interference mode	[0.10 0.10 0.20 0.50 0.10]
Maximum spectral efficiency mode	[0.10 0.15 0.15 0.10 0.50]

Table 2. Results of cognitive radio optimized parameters and objective values obtained by BA for basic CR system

Scenario (Modes of Operation)	Optimized Parameters of bat algorithm					Fitness Score of BA	Values of the Objectives Fulfilled				
	Transmitted Power P (mW)	Modulation Index M	Bandwidth B (Mhz)	TDD (%)	Symbol Rate Rs (Ksps)		Minimum Power Consumed (mW)	Minimum BER	Maximum throughput	Minimum interference	Maximum Spectral Efficiency
Minimizing power mode	02.212	256	2.033	25.0	998.557	0.0265	0.0088	0.2066	1	0.0010	0.9823
Minimizing BER mode	39.890	256	2.102	25.2	998.843	0.0417	0.1589	0.0311	1	0.0032	0.9504
Maximizing throughput mode	19.859	256	2.027	25.1	999.354	0.0178	0.0791 004	0.0553	1	8.4375e-	0.9851
Minimizing interference mode	13.479	256	2.000	25.1	998.255	0.0130	0.0537 009	0.0741	1	1.1723e-	0.9983
Maximizing spectral efficiency mode	17.528	256	2.001	25.1	1000	0.0164	0.0701 005	0.0607	1	3.1251e-	0.9995

Table 3. Comparison of BA, BBO, SA and GA in terms of the fitness score and values of the objectives obtained in each scenario for basic CR system

Scenario	Values	BA	BBO [11]	SA [10]	GA [11]
Min. Power Mode	Fitness Score	0.0265	0.0308	0.0366	0.0547
	Power	0.0088	0.0158	0.0145	0.0175
Min. BER Mode	Fitness Score	0.0417	0.0425	0.0700	0.0867
	BER	0.0311	0.0338	0.0448	0.06281
Max. Throughput Mode	Fitness Score	0.0178	0.0187	0.0238	0.0635
	Throughput	1	1	1	1
Min. Interference Mode	Fitness Score	0.0130	0.015	0.0492	0.0786
	Interference	1.1723e-009	0.0010	4.5974e-10	2.2643e-009
Max. Spec. Eff. Mode	Fitness Score	0.0164	0.0251	0.0194	0.0380
	Spec. Eff.	0.9995	0.9823	0.9992	0.9577

Table 4. Values of the optimized parameters in each scenario along with their respective fitness scores using BA for fading affected multiuser CR system

Scenario (Modes of Operation)	Optimized Parameters of BA										Fitness Score
	P (mw)		M		Rs (Msps)		B (Mhz)		TDD (%)		
	SU1	SU2	SU1	SU2	SU1	SU2	SU1	SU2	SU1	SU2	
Min. Power Mode	0.1000	0.1000	64	64	5.1600	3.8293	6.0000	6.0000	35.90	40.23	0.0815
Min. BER Mode	0.2312	1.1385	64	64	5.1600	5.1600	6.0000	6.0000	33.55	32.60	0.2056
Max. Throughput Mode	0.1000	0.6214	64	64	5.1600	5.1600	6.0000	6.0000	98.08	99.06	0.0752
Min. Interference Mode	0.1000	0.3340	64	64	5.1600	5.1600	6.0000	6.0000	25.00	25.00	0.0540
Max. Spec. Eff. Mode	0.1000	0.6212	64	64	5.1600	5.1600	6.0000	6.0000	25.40	98.78	0.0752

Table 5. Values of the objectives obtained in each scenario using BA for fading affected multiuser CR system

Scenario (Modes of Operation)	Values of the Objectives Fulfilled				
	Min. P	Min. BER	Max. Throughput	Min. Intr.	Max. Spec. Eff.
Min. Power Mode	0.0400	0.5063	1	-7.7419e-011	0.8710
Min. BER Mode	0.2739	0.3563	1	-1.0545e-011	1
Max. Throughput Mode	0.1443	0.4050	1	-1.1593e-011	1
Min. Interference Mode	0.0868	0.4529	1	-7.9125e-011	1
Max. Spec. Eff. Mode	0.1442	0.4050	1	1.7493e-011	1

The convergence characteristics obtained for two of these five objectives are shown in Fig. 2 and Fig. 3.

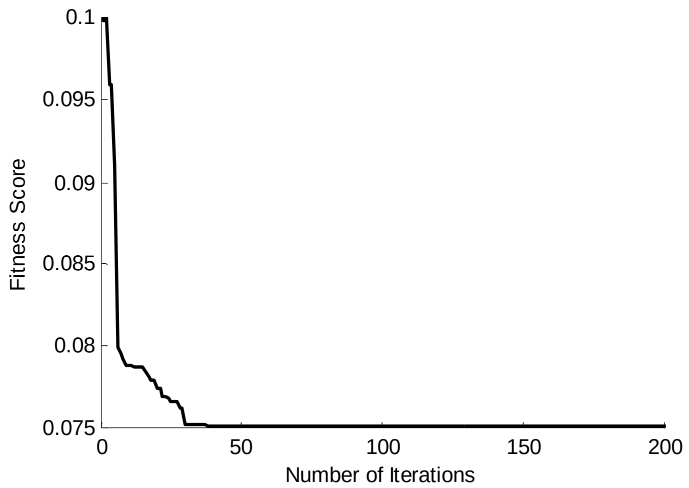

Figure 2. Convergence characteristics for maximum throughput mode

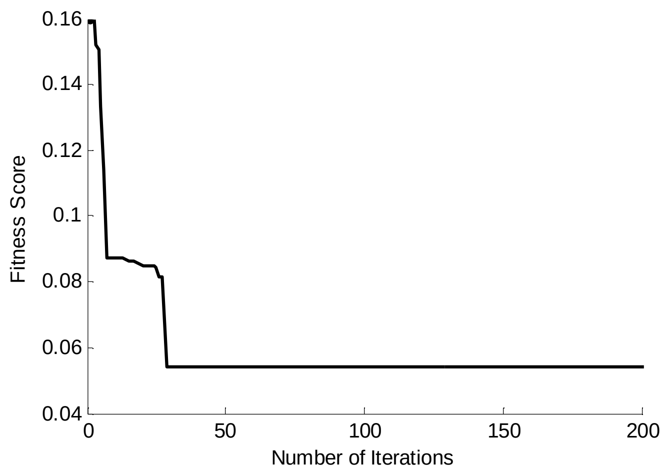

Figure 3. Convergence characteristics for minimum interference mode

Conclusion

To bring the work more close to practicality, a multiuser CR system in TV white space suffering from Nakagami-m fading, using IEEE 802.22 WRAN standard has been optimized successfully by bat algorithm. The efficiency of this algorithm has been first verified by comparing its results with literature results of BBO, SA and GA for a single user basic CR system. Simulations and results show that the transmission parameters (power, modulation index of QAM, bandwidth, time division duplexing and symbol rate) of multiple secondary users of a CR system suffering from fading have also been optimized to satisfy various objectives that are required to achieve desired QoS, considering environmental as well as primary outage constraints. Thus bat algorithm has illustrated to be capable of optimal cognitive radio system designing.

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